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An Empirical study on application of Wilcoxon signed rank test in Statistical Process control when process target is fixed

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Abstract

Many statistical analyses directly or indirectly assume that the population from which the sample was drawn is normally distributed. However there are many cases in which the process distribution is skewed, and such that the normality assumption is not valid. If the process distribution is skewed, the false alarm rate grows larger as the skewness increases. In Statistical Process control also if data do not came from normally distributed population then the use of conventional Shewhart chart is inadequate. Whenever our process target is pre-fixed, for example in packaging industries, we can use Wilcoxon signed rank test statistic. Using this we can try to detect shift from hypothised (assumed) value. We used empirical data to check the successful applicability of Wilcoxon signed rank statistic.

Keywords:

Shewhart chart,
Wilcoxon test,
process target,
non-normal,
empirical data

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1. Introduction:

Every product is made for some specific use, if it serves it completely then it is called of good quality. In different field definition of quality is different, but the concept is same. In manufacturing, products should work as intended with minimum numbers of fault/failures. In management, who wants to see improved production numbers with acceptable quality. Union officials, who want the best conditions and highest pay for employees. Employees, who want consistent work in a safe environment. Customers/users, who want value for their money. In service industry, customer satisfaction is often the primary measure. Conceptually a quality is working on few dimensions like, performance, reliability, durability, serviceability, aesthetics, features, perceived quality, and conformance to the standard. To control these standards we use SPC technique. We know in many industries the process mean of respected variable is unknown. For example diameter of rings, diameter of bearing, voltage, width, height of screw, etc. The central lines for these variables are set on the sample average, by controlling dispersion. But whenever our process mean is prefixed, before starting up the process, for example in packaging industries, the net weight. In such problem our target net weight is prefixed, and instead of fixing this targeted value as central line, if we fix sample average as central line, then our process mean can be distract from target.

2. Objective of the study

i. Which statistical control chart is more efficient when process mean is fixed?

ii. Which SQC chart is advisable to use, when data does not follows normal distribution? Or we do not able to check its Normality due to small sample size.

As per our objective we found that for unknown distribution and known process target Wilcoxon signed rank statistics is appropriate. To check its efficiency and applicability we collected 5 observations of 25 machines, packing milk in pouches of 500ml (519gms).

3. A review of Literature on Non-parametric charts for variable

Distribution-free or nonparametric control charts can be useful in a variety of statistical process control problems. A key advantage of distribution-free charts is that the user does not need to assume any particular distribution (such as the normal distribution) for the underlying process and the in-control probability calculations and associated conclusions remain valid for any continuous distribution. This distribution robustness could be an advantage, particularly, in start-up situations where we usually do not have knowledge of the underlying distribution.

Young H. Chun (2000) constructed a nonparametric control chart for symmetric process. Fahad S. Altukife, Jay R. Schaffer(2001) Presented a new nonparametric charting scheme based on order statistics. S. Chakraborti, P. van der Laan and M. A. van de Wiel (2004) examined median based control chart proposed by Janacek-Meikle and connected this with 'control median statistic' of Mathisen (1943) and developed precedence control chart. Saad.T. Bakir (2004) proposed Wilcoxon signed-rank statistic, instead of Shewhart control chart. S. Chakraborti1 and S. Eryilmaz (2007) considered Shewhart-type distribution-free control charts for the known in control median of a continuous process distribution based on the Wilcoxon signed rank statistic and some runs type rules. Nandini Das (2009) summarized the different nonparametric control charts for controlling location from a literature survey, viz. control charts based on the sign test, control charts based on the Hodges–Lehmann estimator and control charts based on the Mann–Whitney statistic and compared their efficiency to detect the shift in location while in out of the control state under different situations and identified the best method under the prevailing situation. Nandini Das (2008) developed non-parametric control chart for controlling variability based on Conover's squared rank test for variance. And Nandini Das and Anik Bhattacharya (2008) used this chart with numerical example. Nandini Das (2008) adopted Mood test and Tukey test, the tests of significance between two population variances for monitoring variability in control charts. Subhabrata Chakraborti, Mark A Van de weil (2008) used Mann Whitney statistic for location of the process against the Shewhart's $\overline{X}$ chart. Chunguang Zhou, Changliang Zou, Yujuan Zhang, Zhaojun Wang (2009) used nonparametric, Mann Whitney statistics to check significance of change in median for detecting shifts in the mean of a process, where the nominal value of the mean is unknown but some historical samples are available. S.K.Khilare, D.T.Shirke (2010) checked properties of synthetic control chart using sign test. Saad Bakir (2012) proposed a nonparametric Shewhart type control chart for monitoring a broad change in a process probability distribution. To develop the chart, he assumed the availability of a reference sample taken when the process was operating in statistical control. The charting statistic is a modified version of the two-sample Kolmogorov-Smirnov test statistic where the difference of the reference and test empirical distribution functions is maximized only over the reference sample values.

4. Basic requirements and Procedure to find Wilcoxon Statistic:

I. Assumptions to be fulfilled for this statistic:

1. The distribution of each $D_i=X_i-\mu$ is symmetric and continuous. Where μ=process target.
   (Symmetricity for Di, we can check using Sign Test. For sample size n=5 and under assumption of symmetricity, probability of no. of –ve(or +ve) sign=0.5. and at 0.0027 level of significance for two tailed alternative hypothesis critical value is 0.)
2. The D_i's are mutually independent. (Xi's are independent so $D_i=X_i-\mu$ are also independent.)
3. The measurement scale of the D_i's is at least interval. (Weight measured on ratio scale.)

II. Test Procedure and statistic:

Wilcoxon proposed the test for equality of one population location parameter with the hypothised value. The sample of size n having observations $x_1, x_2, \ldots, x_n$.

Compute Xi- μ, i=1,2,…n. rank the absolute differences |Xi- μ| in ascending order and then give the ranks the sign of their corresponding differences. Let W+ be the sum of positive ranks and W- be the absolute value of sum of negative ranks, and let

$$W=\min(W+,W-) \qquad \ldots\ldots\ldots(1)$$

III. Procedure for constructing control chart:

Step-1: Collect k samples of size n.

Step-2: For each sample compute W using test procedure.

Step-3: We have continuous random variable, and sample size n=5.

Step-4: As in Wilcoxon test statistic we are comparing minimum sum of rank of particular sign, either positive or negative, i.e. W+ or W-, with critical value. So obviously in control chart we would have only lower control limit, not the upper control limit. As we are comparing minimum no. of sum with lower critical value. To overcome with this we found $\psi = 2*W - n(n+1)/2$ (for more detail see Saad Bakir-2004) to plot in control chart. For a two-sided Control Chart, a lower control limit LCL<0 and an upper control limit UCL>0 are chosen to give a certain false alarm rate or in-control ARL=1/p, where p is probability of signal=P[$\Psi \notin$ (LCL,UCL)] for a two sided chart. Choosing LCL= -UCL results in a symmetric two-sided chart. As $\psi \in \left(-\frac{n(n+1)}{2}, \frac{n(n+1)}{2} \right)$

IV. Run length Distribution for different n, sample sizes:

A popular measure of chart performance is the expected value of the run length (the number of samples or subgroups that need to be collected before the first out of control signal is given by a chart is a random variable called the run length.) distribution called the average run length (ARL). By definition, the run length is a positive integer valued random variable, so the ARL loses much of its attractiveness as a typical summary if the distribution is skewed (as is often the case).

The values of UCL for different probabilities are represented in table-1. Values of various control limits for different probabilities and ARL are given in Table-1, for sample size 4, 5, 6.

The run lengths of a two-sided Shewhart signed-rank chart is given by the random variable L= $\min_t \{t : \psi_t \le LCL \ or \ \psi_t \ge UCL\}$

V. Calculation:

To implement the proposed chart in practice, a practitioner needs to know the values of the control limits that guarantee a specific in-control average run length and/or a false alarm rate for the chart. Set the control chart limits for sample size n, as given in step-4 mentioned above, and probability of false alarm p=0.0027, as for 3σ control limits of $\overline{X}$ chart the probability of false alarm is 0.0027.

Upper control limit (UCL) =0.00135 quantile = -15

Central line = 0.5 quantile=0

Lower control limit (LCL)= 0.99865 quantile =15, for n=5

Plot ψ_t values in control chart. If any point goes beyond the limit it will indicate that the process is out of control with respect to variability.

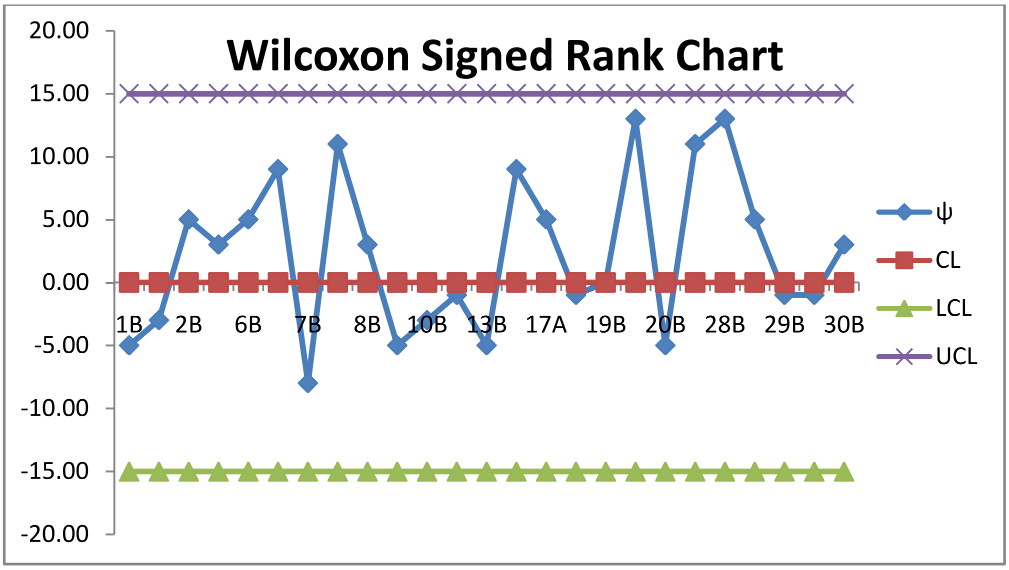

Chart-1

Here each Wilcoxon Statistic is falling within control limits. So we can say that the process is under control with respect to process target.

Similarly if we draw the Traditional Shewhart control chart for $\overline{X}$ with s, then for 0.0027 central line and control limits are

UCL= $\overline{\overline{X}} + A_1\overline{s}$ =528.8

CL= $\overline{\overline{X}}$ =521.67

LCL= $\overline{\overline{X}} - A_1\overline{s}$ =513.15

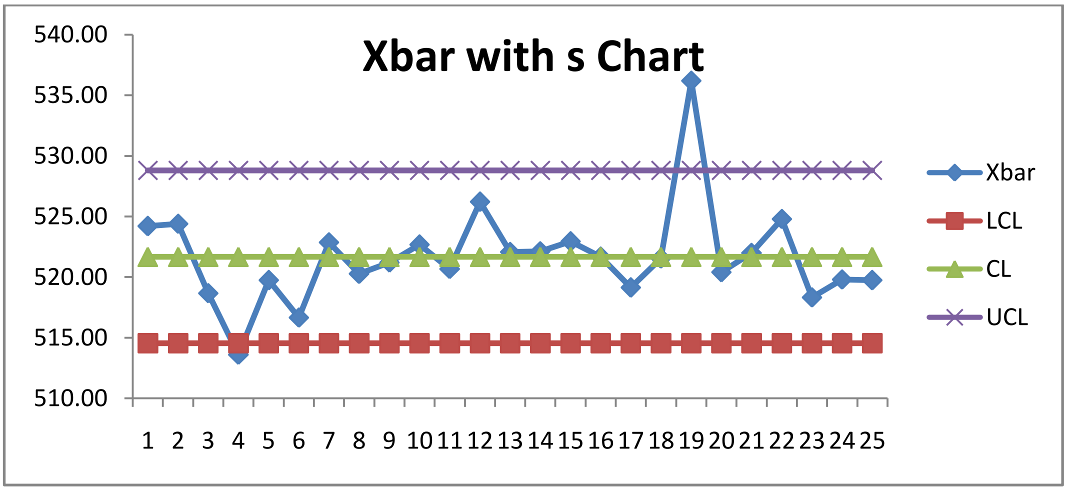

Chart-2

From this chart we can clearly say that 2 sample points are falling outside of control limits. Which may happened due to non-normality of data.

5. Conclusion:

If without checking the assumption of Normality, required for traditional Shewhart control chart, we draw it. Then it can increase the false alarm rate. And can show the control process is running out of control. With the same degree of false alarm Wilcoxon Shows that the process is under control.

Also in Wilcoxon we measured the deviation of observations from the prefixed process target. But in traditional Shewhart $\overline{X}$ chart we are finding deviation from estimated value of the process target $\left(s = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n-1}} \right)$. That is why also it is misleading us, by increasing variation from its estimated mean, instead of actual or known mean.

Also when sample size is small, we can not apply Xbar chart under assumption of Normality, but there is not such restriction for Wilcoxon.

Appendices

Table-1

	n=4		n=5		n=6	
UCL=-LCL	ARL	p	ARL	p	ARL	p
7	3.6670	0.2727	2.2857	0.4375	1.7778	0.5625
9	5.5006	0.1818	3.2000	0.3125	2.2857	0.4375
11	>11.0011	<0.0909	5.3333	0.1875	3.2000	0.3125
13			8.0000	0.125	4.5714	0.21875
15			>16.0000	<0.0625	6.4000	0.15625
17					10.6838	0.0936
19					16.0000	0.0625
21					>32.0000	<0.03125

Table-2

machine	**S-1**	**S-2**	**S-3**	**S-4**	**S-5**	**Xbar**
1A	521.33	523.67	527.67	522.67	525.67	524.20
1B	523.33	521.67	534.67	519.33	523.00	524.40
2A	516.33	518.33	521.00	518.67	519.00	518.67
2B	518.33	519.67	521.67	488.00	520.33	513.60
7A	520.00	518.33	521.33	520.00	519.00	519.73
7B	517.00	516.67	518.67	519.00	512.00	516.67
8A	525.67	524.00	522.33	523.67	518.67	522.87
10A	518.00	520.67	518.00	520.33	524.33	520.27
10B	522.67	524.00	520.00	520.00	519.33	521.20
13A	522.00	521.00	524.33	521.67	524.33	522.67
13B	523.00	519.33	518.33	521.00	521.67	520.67
17A	522.00	514.67	538.33	529.33	526.67	526.20
17B	520.67	514.00	528.67	525.67	521.33	522.07
18A	526.00	523.00	518.33	525.00	518.33	522.13
18B	525.33	522.33	521.67	516.67	528.67	522.93
19A	522.67	520.00	520.67	523.33	521.67	521.67
19B	518.33	514.00	520.67	523.00	519.67	519.13

20A	524.33	521.33	517.67	517.67	526.67	521.53
28B	531.33	534.67	526.67	527.67	560.67	536.20
29A	519.67	518.00	521.00	521.33	522.00	520.40
30A	518.00	518.33	525.33	521.33	527.00	522.00
30B	524.67	529.67	525.00	517.67	527.00	524.80
29B	516.67	515.00	509.00	524.00	527.00	518.33
30A	518.00	517.33	517.00	520.33	526.33	519.80
30B	512.00	516.67	521.67	520.67	527.67	519.73

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